

# Repulsive Short Range Three-Nucleon Interaction

S. A. Coon<sup>1</sup>, M. T. Peña<sup>2</sup>, and D. O. Riska<sup>3</sup>

<sup>1</sup> *Physics Department, New Mexico State University, Las Cruces, New Mexico 88033, USA*

<sup>2</sup> *Centro de Fisica Nuclear, 1699 Lisboa and Instituto Superior Técnico, 1096 Lisboa, Portugal*

<sup>3</sup> *Department of Physics, University of Helsinki, 00014 Finland*

## Abstract

The three nucleon interaction that arises from pion and the effective scalar and vector meson exchange components of the nucleon-nucleon interaction is constructed and shown to be repulsive. Using several wavefunction models we show that this interaction reduces the calculated binding energy of the trinucleons by about 200 keV. The contributions of intermediate  $N(1440)$  resonances to this three nucleon interaction and of the  $D$ -state component in the trinucleon model are estimated, and shown to be small.

Submitted to The Physical Review C

Research Institute for Theoretical Physics  
University of Helsinki Preprint HU-TFT-95-11

## 1. Introduction

Although the nuclear three-nucleon interaction (TNI) is very weak in comparison to the two-nucleon interaction [1], it nevertheless has been found that the binding energies of the bound three- and four-nucleon systems cannot be understood without taking into account the attraction caused by the TNI [2]. The main component of the nuclear TNI is that associated with two-pion exchange, which arises from, for the major part, pion rescattering through an intermediate virtual  $\Delta_{33}$  resonance. The standard model for the two-meson exchange component of the TNI is the so-called Tucson-Melbourne model, which also includes  $\rho$ -meson exchange in addition to pion exchange [3, 4]. It has been found that when realistic models for the nucleon-nucleon interaction are employed, the Tucson-Melbourne  $\pi$ -exchange model for the TNI leads to an overbinding of a few hundred keV in the trinucleons [5] and of 2-4 MeV in the case of the alpha particle [6]. To compensate for this overbinding an additional repulsive spin-independent phenomenological TNI of short range has been proposed [7]. On the other hand, the Tucson-Melbourne model (*with both  $\pi$ - and  $\rho$ -exchange*) has recently been shown to give the correct binding of the tri-nucleon with the Tucson-Melbourne meson-baryon-baryon vertex functions (“form factors”) [8]. These form factors, however, differ in their short range behavior from those of the (older) realistic models of the nucleon-nucleon interaction. While this form factor discrepancy [9] is further studied in the case of the NN interaction [10], there remains a need for further investigation of the short range aspects of the TNI.

There is no known dynamical mechanism that would lead to a spin-independent short range TNI. The short range three-nucleon interaction that arises from scalar and vector meson exchanges with an intermediate nucleon-antinucleon pair were considered in ref.[11], but were found to be both insignificantly small and spin dependent. We here consider another set of related three-nucleon interactions - those that arise from pion and “effective” scalar and vector meson exchanges and that involve excitation of intermediate nucleon-antinucleon pairs and  $N(1440)$  resonances (Fig. 1). The presence and form of the former (Fig. 1a) - and more important one of these interactions - is implied by the pion and scalar and vector meson exchange components of the nucleon-nucleon interaction, and can be derived directly from a given complete model for the two-nucleon interaction. The derivation is es-

pecially straightforward if the nucleon-nucleon interaction has explicit scalar and vector meson exchanges with associated meson-baryon-baryon vertex functions, but the former TNI can also be constructed from “effective” scalar and vector meson exchanges obtained from any complete energy-independent model of the nucleon-nucleon interaction [12]. The magnitude of the latter TNI (Fig. 1b) is less certain because of the wide uncertainty in the meson-nucleon- $N(1440)$  (“Roper resonance”) coupling strengths. Both of these sets of three-nucleon interactions nevertheless have a very simple form, although spin dependent, and when combined provide an amount of additional repulsion that approximately corresponds to that required and hitherto ascribed to the purely phenomenological spin-independent short range TNI.

In section 2 of this paper we derive the effective  $\pi$ -scalar and  $\pi$ -vector meson exchange three nucleon interactions which arise from excitation of intermediate nucleon-antinucleon pairs and show how the potentials which describe these interactions can be constructed from realistic models for the nucleon-nucleon interaction. It is this construction method which distinguishes the TNI’s of the present paper from the Brown-Green [1], Tucson-Melbourne [3, 4], Brazil [13], and the Fujita-Miyazawa [14] TNI’s (the last being extensively used in a program of studies with light nuclear systems [15]) The latter TNI’s did not attempt such a tight connection with a nucleon-nucleon interaction but relied on other aspects of hadronic phenomenology for motivation and parameter fixing. In section 3 we derive the corresponding three nucleon interactions that arise from excitation of  $N(1440)$  resonances on the intermediate nucleon and derive the corresponding coupling constants from the partial decay widths. In section 4 we present numerical results for the contribution to the binding energy of the trinucleons, which arises from these interactions using oscillator and Malfliet-Tjon [16] wavefunctions as well as Paris and Bonn OBEPQ [8, 17] wavefunctions. An estimate that indicates that the  $D$ -state component of the trinucleon bound state contributes only an insignificant contribution to the net matrix element of the  $\pi$ -short range three-nucleon interaction is presented in section 5. Finally section 6 contains a concluding discussion. The partial wave decomposition of the TNI is described in the Appendix.

## 2. Pion-scalar and -vector meson exchange three-nucleon interactions

Realistic models for the nucleon-nucleon interaction as eg. the Bonn [18], Nijmegen [19], and Paris [20] models are based on meson exchange models, in which the Lorentz invariant on-shell nucleon-nucleon scattering amplitude is constructed from phenomenological meson-nucleon Lagrangians, and then used as an off-shell kernel in a wave-equation. In practice a nonrelativistic “adiabatic” approximation is also involved [21, 22]. The important point is that a given model for the on-shell amplitude that is extrapolated off shell in this way automatically implies a model for the  $NN \rightarrow NNN\bar{N}$  amplitude as well. This amplitude is the central component in the three-nucleon interactions that arise from excitation of virtual nucleon-antinucleon pairs on the intermediate nucleon (Fig. 1a). Thus any meson exchange model for the nucleon-nucleon interaction will by construction imply the presence of three-nucleon interactions of this type, which formally can be derived directly from the  $NN$  interaction model without any need for further assumptions.

We shall here consider the  $\pi$ -scalar and  $\pi$ -vector meson exchange TNI’s of this form. An important part of the motivation for this is the recent observation that the off shell  $\pi + N \rightarrow N + \text{scalar}$  and  $\pi + N \rightarrow N + \text{vector}$  amplitudes are “almost” observable, in that they successfully describe most of the cross section for the reaction  $pp \rightarrow pp\pi^0$  near threshold [23]. Thus the theoretical model for these amplitudes can be viewed as having a good empirical foundation. In the derivation of these three-nucleon amplitudes we shall use the usual phenomenological scalar (“ $\sigma$ ”, “ $a_0$ ”) and vector meson (“ $\omega$ ”, “ $\rho$ ”) Lagrangians, but in the end will replace the pure boson exchange interactions by the corresponding “effective” scalar and vector meson exchange interactions that form the most important short range components of the nucleon-nucleon interaction.

To construct the  $\pi$ -scalar (“ $\sigma$ ”)-meson three-nucleon interaction that corresponds to the Feynman diagram in Fig. 1a we employ the  $\pi NN$  and  $\sigma NN$  couplings

$$\mathcal{L}_{\pi NN} = i \frac{f_{\pi NN}}{m_\pi} \bar{\psi} \gamma_5 \gamma_\mu \partial_\mu \vec{\phi}_\pi \cdot \vec{\tau} \psi, \quad (2.1a)$$

$$\mathcal{L}_{\sigma NN} = g_\sigma \bar{\psi} \phi_\sigma \psi. \quad (2.1b)$$

Here  $\vec{\phi}_\pi$  is the isovector pion and  $\phi_\sigma$  the isoscalar scalar meson field and  $f_{\pi NN}$  and  $g_\sigma$  the corresponding coupling constants.

The three-nucleon interaction that corresponds to the Feynman diagram in Fig. 1a is obtained by retaining only the negative energy part of the fermion propagator for the intermediate nucleon and adding the term with the pion and scalar meson couplings in reversed order. In the actual construction of the operator we exploit the fact that the nucleons are nearly on shell, and use the Dirac equation to simplify the algebra. In this way a contact term operator arises, which has to be retained along with the pair term in the three- nucleon interaction operator. The resulting three-nucleon interaction has the simple form

$$V_{\pi\sigma} = \frac{g_\sigma^2}{m_N} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi \vec{\sigma}^2 \cdot \vec{k}_\sigma}{(k_\pi^2 + m_\pi^2)(k_\sigma^2 + m_\sigma^2)} \vec{\tau}^1 \cdot \vec{\tau}^2 + (\text{permutations}). \quad (2.2)$$

Here the direction of the meson momenta are taken so that they point away from the intermediate nucleon, that is indicated by the superscript 2 on the spin and isospin operators. The symbol “permutations” stands for first adding a term, in which the nucleon coordinates 1 and 2 are exchanged and then taking into account the additional (4) terms in which the intermediate nucleon-nucleon pair is excited on nucleon 1 and 3 in turn.

In order to make this expression for the pion-scalar TNI consistent with a realistic model for the nucleon-nucleon interaction it is natural to replace the term  $-g_\sigma^2/(k_\sigma^2 + m_\sigma^2)$  in the expression (2.2) by the corresponding general “effective” isospin independent scalar component  $v_S^+(\vec{k}_\sigma)$  of the nucleon-nucleon interaction, which may be constructed using the method of ref. [12]. In a similar way it is natural to replace the term  $(f_{\pi NN}/m_\pi)^2/(k_\pi^2 + m_\pi^2)$  by the effective isospin dependent pseudoscalar exchange potential  $v_P^-(\vec{k}_\pi)/4m_N^2$  of the nucleon-nucleon interaction. In this way the short range modifications - i.e. form factors - of the simple meson exchange interactions are determined by the corresponding components of the nucleon-nucleon interaction model.

Specifically, the method of ref. [12] rewrites a nonrelativistic nucleon-nucleon potential model on the energy shell in terms of five nonrelativistic spin amplitudes which can be viewed as nonrelativistic limits of five relativistic Fermi invariants. For the construction of the TNI's in this paper we need the isospin independent (+) and isospin dependent (-) Fermi invariants: scalar ( $S$ ), pseudoscalar ( $P$ ), and vector ( $V$ ). The Fermi invariant potential coefficients  $v_j^\pm(\vec{k})$ ,  $j = A, P, V$  are obtained as linear combinations of the nonrelativistic components of a given potential. The Fermi invariant potential coefficients  $v_j^\pm$  are functions of  $k^2$  only which means that the underlying interactions have no energy dependence. The procedure and results for carrying out this program for a potential (such as Paris), which has a short range behavior determined in coordinate space, are displayed in [12]. If the potential is already expressed in terms of relativistic invariants corresponding to the exchange of scalar, vector, and pseudoscalar bosons (such as the Bonn or Nijmegen potentials) the procedure amounts to the replacement (for example)

$$\frac{(f_{\pi NN}/m_\pi)^2}{m_\pi^2 + k_\pi^2} \rightarrow \frac{v_P^-(\vec{k}_\pi)}{4m_N^2} \rightarrow \frac{(f_{\pi NN}/m_\pi)^2}{k_\pi^2 + m_\pi^2} \left( \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + k_\pi^2} \right)^2, \quad (2.2a)$$

where the meson-nucleon-nucleon vertex  $(\Lambda_\pi^2 - m_\pi^2)/(\Lambda_\pi^2 + k_\pi^2)$ , is of the form chosen for the Bonn OBEPQ potential used in our numerical investigations. In either case, the short range behavior of the TNI so constructed is fully determined by the short range behavior of the corresponding nucleon-nucleon interaction, which is the issue at hand.

In the case of an isospin-1 scalar meson exchange ( $a_0$ -channel) the TNI that corresponds to the expression (2.2) is

$$V_{\pi a} = \frac{g_a^2}{m_N} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi}{(k_\pi^2 + m_\pi^2)(k_a^2 + m_a^2)} \{ \vec{\sigma}^2 \cdot \vec{k}_a \vec{\tau}^1 \cdot \vec{\tau}^3 + 2i \vec{\sigma}^2 \cdot \vec{P}_2 \vec{\tau}^1 \cdot \vec{\tau}^2 \times \vec{\tau}^3 \}. \quad (2.3)$$

Here  $g_a$  is the  $a_0 NN$  coupling constant and  $m_a$  and  $\vec{k}_a$  the mass and momentum of the exchanged  $a_0$  respectively. In view of the smallness of the nucleon momenta in the bound states we shall not consider the non-local term in this TNI, which contains the momentum  $\vec{P}_2 = (\vec{p}_2 + \vec{p}'_2)$  of the intermediate

nucleon. In order to make the  $\pi a_0$  TNI (2.3) consistent with the models for the nucleon-nucleon interaction we shall replace the simple  $a_0$  exchange interaction in (2.3) by the corresponding isospin dependent scalar exchange component of the nucleon-nucleon interaction:  $g_a^2/(m_a^2 + k_a^2) \rightarrow -v_S^-(\vec{k}_a)$ .

To construct the  $\pi\omega$  TNI we employ the  $\omega NN$  Lagrangian

$$\mathcal{L} = ig_\omega \bar{\psi} \gamma_\mu \omega_\mu \psi, \quad (2.4)$$

where  $g_\omega$  is the  $\omega NN$  coupling constant in addition to (2.1a). The resulting TNI potential is

$$V_{\pi\omega} = -\frac{g_\omega^2}{m_N} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi \vec{\sigma}^2 \cdot \vec{k}_\omega}{(k_\pi^2 + m_\pi^2)(k_\omega^2 + m_\omega^2)} \vec{\tau}^1 \cdot \vec{\tau}^2 + (\text{permutations}). \quad (2.5)$$

This interaction, which arises from the charge component of the  $\omega$ -field is similar in form to the  $\pi\sigma$  TNI (2.2), but has the opposite sign. As in the case of the two-nucleon interaction there will therefore be a strong partial cancellation between the  $\pi\sigma$  and  $\pi\omega$  three-nucleon interactions, such that the  $\pi\sigma$  typically is the stronger interaction, because of the somewhat longer (intermediate) range of the effective scalar meson exchange interaction.

The  $\pi\rho$  exchange TNI has one component that arises from the charge component and one that arises from the spatial component. The first one of these has the form

$$V_{\pi\rho}^C = -\frac{g_\rho^2}{m_N} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi \vec{\sigma}^2 \cdot \vec{k}_\rho}{(k_\pi^2 + m_\pi^2)(k_\rho^2 + m_\rho^2)} \vec{\tau}^1 \cdot \vec{\tau}^3, \quad (2.6)$$

and the expression for the latter is

$$V_{\pi\rho}^S = -i \frac{g_\rho^2}{m_N} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi}{(k_\pi^2 + m_\pi^2)(k_\rho^2 + m_\rho^2)} \sigma^2 \cdot [2\vec{P}_3 + i\vec{\sigma}^3 \times \vec{k}_\rho] \vec{\tau}^1 \cdot \vec{\tau}^2 \times \vec{\tau}^3. \quad (2.7)$$

The local part of the  $\pi\rho$  exchange three nucleon interaction  $V_{\pi\rho}^S$  is referred to as the “ $\pi\rho$  Kroll-Rudermann interaction” [4] or the “seagull” [13] in the

literature, and has been considered in the trinucleon before [8, 24, 25]. In the treatment of the Tucson-Melbourne TNI (an expansion of the  $\rho N \rightarrow \pi N$  amplitude)  $V_{\pi\rho}^C$  is small because of a near cancellation between the intermediate nucleon-antinucleon part and the  $\rho$  analog Fubini-Furlan-Rossetti contribution to pion photoproduction. The isoscalar  $V_{\pi\rho}^C$  corresponds to the remaining lead term in eqs. 2.13b and 2.14a of [4a]. It was estimated in nuclear matter in [4a], derived and then neglected altogether in the Brazil TNI [13], and is evaluated in the trinucleon for the first time in the present study. To make the present expressions for the  $\pi\omega$  and  $\pi\rho$  exchange three-nucleon interactions consistent with the nucleon-nucleon interaction model we shall replace the bare vector meson interactions  $g_\omega^2/(k_\omega^2 + m_\omega^2)$  and  $g_\rho^2/(k_\rho^2 + m_\rho^2)$  with the corresponding isospin independent ( $v_V^+(\vec{k}_\omega)$ ) and isospin dependent ( $v_V^-(\vec{k}_\rho)$ ) vector exchange components of the nucleon interaction as suggested in ref. [12].

### 3. Intermediate N(1440) resonances

Excitation of virtual  $N(1440)$  (Roper) resonances on the intermediate nucleon also contributes a weak but still significant  $\pi$ -scalar and  $\pi$ -vector meson exchange three nucleon interaction (Fig. 1b). Naturally a contribution to the  $\pi\pi$  three-nucleon interaction also arises from intermediate  $N(1440)$  excitation, but this is effectively included in the Tucson-Melbourne TNI, which is based on an off shell extrapolation of the complete  $\pi N$  scattering amplitude.

To construct the  $\pi\sigma$  and  $\pi\omega$  three nucleon interactions that are associated with intermediate  $N(1440)$  resonance excitation we employ the effective Lagrangians

$$\mathcal{L}_{\pi NN^*} = i \frac{f_\pi^*}{m_\pi} \bar{\psi}_* \gamma_5 \gamma_\mu \partial_\mu \vec{\phi} \cdot \vec{\tau} \phi + \text{h.c.}, \quad (3.1a)$$

$$\mathcal{L}_{\sigma NN^*} = g_\sigma^* \bar{\psi}_* \phi_\sigma \psi + \text{h.c.}, \quad (3.1b)$$

$$\mathcal{L}_{\omega NN^*} = i g_\omega^* \bar{\psi}_* \gamma_\mu \omega_\mu \psi + \text{h.c.}. \quad (3.1c)$$

Here  $\psi_*$  denotes the Roper resonance spinor field and  $f_\pi^*$ ,  $g_\sigma^*$  and  $g_\omega^*$  are the



$\pi NN^*$ ,  $\sigma NN^*$  and  $\omega NN^*$  coupling strengths,  $N^*$  being an abbreviation for the  $N(1440)$ . The expressions for the  $\pi\sigma$  and  $\pi\omega$  TNI potentials that are associated with intermediate  $N(1440)$  excitation can then be derived in a straightforward way, the results being

$$V_{\pi\sigma}^* = -\frac{2g_\sigma g_\sigma^*}{m^* - m_N} \frac{f_{\pi NN} f_\pi^*}{m_\pi^2} \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi \vec{\sigma}^2 \cdot \vec{k}_\pi}{(k_\pi^2 + m_\pi^2)(k_\sigma^2 + m_\sigma^2)} \vec{\tau}^1 \cdot \vec{\tau}^2 + (\text{permutations}), \quad (3.2a)$$

$$V_{\pi\omega}^* = \frac{2g_\omega g_\omega^*}{m^* - m_N} \frac{f_{\pi NN} f_\pi^*}{m_\pi^2} \frac{\vec{\sigma}^1 \cdot \vec{k}_\pi \vec{\sigma}^2 \cdot \vec{k}_\pi}{(k_\pi^2 + m_\pi^2)(k_\omega^2 + m_\omega^2)} (\vec{\tau}^1 \cdot \vec{\tau}^2). \quad (3.2b)$$

Here  $m^*$  is the mass of the  $N(1440)$  resonance. It is worth noting that the terms that depend on the pion momentum  $\vec{k}_\pi$  in these three-nucleon interactions have the same form as the one-pion exchange interaction between two nucleons.

The  $\pi NN^*$  coupling constant  $f_\pi^*$  may be calculated from the  $N^* \rightarrow N\pi$  partial decay width as

$$\frac{f_\pi^{*2}}{4\pi} = \frac{m^* m_\pi}{(m^* + m_N)p(E_N - m_N)} \Gamma(N^* \rightarrow N\pi). \quad (3.3)$$

Here  $p$  is the nucleon momentum and  $E_N$  the nucleon energy in the rest frame of the decaying  $N^*$ . With the  $N\pi$  branching ratio of the total decay width 350 MeV being 60% we obtain  $f_\pi^*/4\pi \simeq 0.031$ , which is somewhat less than one half of the corresponding value  $f_{\pi NN}^2/4\pi \simeq 0.08$  for the  $\pi NN$  coupling strength.

The determination of the  $\sigma NN^*$  and  $\omega NN^*$  coupling strength is associated with considerably larger uncertainties. To obtain an estimate for the  $\sigma NN^*$  coupling constant we calculate it from the decay width for  $N^* \rightarrow N(\pi\pi)_{S\text{-wave}}^{I=0}$  as

$$\frac{g_\sigma^{*2}}{4\pi} = \frac{m^*}{p(E_N + m_N)} \Gamma(N^* \rightarrow N(\pi\pi)_{S\text{-wave}}^{I=0}). \quad (3.4)$$

Here we assume that all of the  $I = 0$   $S$ -wave part of the  $\pi\pi$  continuum can be interpreted as a broad effective  $\sigma$ -meson. The branching ratio for this

decay channel is 5-15%. Assuming  $m_\sigma = 410$  MeV at the midpoint between the  $\pi\pi$  threshold and kinematical phase space cutoff we obtain

$$\frac{g_\sigma^{*2}}{4\pi} \simeq 0.1 \quad (3.5)$$

for  $\Gamma(N^* \rightarrow N(\pi\pi)_{S\text{-wave}}^{I=0}) = 35$  MeV. This value for  $g_\sigma^{*2}/4\pi$  is expected to have an uncertainty of about a factor 2.

As the  $N^*$  cannot decay into a  $N\omega$  state, the coupling constant  $g_\omega$  cannot be determined directly from empirical data. We shall here assume that  $g_\omega^*/g_\omega = g_\sigma^*/g_\sigma$  as suggested by the constituent quark model. In the Bonn boson exchange model OBEPQ for the nucleon-nucleon interaction this ratio is 1.55 [18]. This would then suggest that

$$\frac{g_\omega^{*2}}{4\pi} = 0.24, \quad (3.6)$$

a value with which a substantial uncertainty margin also has to be associated.

The bare meson exchange potentials in the TNI's (3.2a) and (3.2b) will be modified at high values of momentum transfer by shorter range dynamics in the same way as the  $NN$  interaction. To describe this short range modification in a way that is consistent with that of the  $NN$  interaction we shall introduce the same vertex factors as in the Bonn boson exchange model OBEPQ for the  $NN$  interaction [18] by means of the substitutions

$$\frac{1}{m_\pi^2 + k_\pi^2} \rightarrow \frac{1}{k_\pi^2 + m_\pi^2} \left( \frac{\Lambda_\pi^2 - m_\pi^2}{\Lambda_\pi^2 + k_\pi^2} \right)^2, \quad (3.7a)$$

$$\frac{1}{m_\sigma^2 + k_\sigma^2} \rightarrow \frac{1}{k_\sigma^2 + m_\sigma^2} \left( \frac{\Lambda_\sigma^2 - m_\sigma^2}{\Lambda_\sigma^2 + k_\sigma^2} \right)^2, \quad (3.7b)$$

$$\frac{1}{m_\omega^2 + k_\omega^2} \rightarrow \frac{1}{k_\omega^2 + m_\omega^2} \left( \frac{\Lambda_\omega^2 - m_\omega^2}{\Lambda_\omega^2 + k_\omega^2} \right)^2. \quad (3.8b)$$

For the form factor mass scale parameters we use the values  $\Lambda_\pi = 1.3$  GeV/ $c^2$ ,  $\Lambda_\sigma = \Lambda_\omega = 2.0$  GeV/ $c^2$ .

#### 4. Numerical estimates

We shall here estimate the contribution of the  $\pi$ -scalar and  $\pi$ -vector exchange TNI to the binding energy of the trinucleons with a pure  $S$ -state wavefunction model. In the first estimates presented, the orbital part of the wavefunction is described by the harmonic oscillator and three-channel Malfliet-Tjon I-III model wavefunction [16]. The use of the harmonic oscillator model is motivated by the fact that the resulting matrix elements can be reduced to quadrature of very simple expressions, which allows the qualitative features to be illuminated. The numerical values for the matrix elements of the three-nucleon interactions considered here that are obtained with the harmonic oscillator and Malfliet-Tjon wave function models turn out to be very similar.

In the case of a wavefunction with only a completely symmetric  $S$ -state component the radial matrix element may be expressed as

$$\langle V_3 \rangle = \int \frac{d^3\tau}{(2\pi)^3} \int \frac{d^3v}{(2\pi)^3} g(\vec{\tau}, \vec{v}) \phi_0^\dagger V_3(\vec{\tau}, \vec{v}) \phi_0. \quad (4.1)$$

Here  $\phi_0$  is the totally antisymmetric spin-isospin vector and  $\vec{\tau}$  and  $\vec{v}$  are differences of nucleon Jacobi coordinates, which are related to the meson momenta in the three-nucleon interactions as

$$\vec{k}_\pi = (\vec{\tau} + \frac{\vec{v}}{2}), \quad \vec{k}_{\sigma,\omega} = \vec{v}. \quad (4.2)$$

In (4.1) the function  $g(\vec{\tau}, \vec{v})$  is the Fourier transform of the nucleon density function in coordinate space, which in the case of the harmonic oscillator model takes the form

$$g(\vec{\tau}, \vec{v}) = e^{-\tau^2/2\alpha^2} e^{-3v^2/8\alpha^2}, \quad (4.3)$$

$\alpha$  being the oscillator parameter  $\sqrt{m\omega_0}$  for which we use the value  $0.60 \text{ fm}^{-1}$ .

The oscillator model leads to the following expression for the matrix element of the  $\pi\sigma$  three-nucleon interaction (2.2) in  ${}^3H$  and  ${}^3He$ :

$$\langle V_{\pi\sigma} \rangle = \frac{3}{2\pi^4} \frac{g_\sigma^2}{m_N} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 \int_0^\infty dv e^{-v^2/2\alpha^2} \frac{v^3}{v^2 + m_\sigma^2}$$

$$\int_0^\infty dk e^{-k^2/2\alpha^2} \frac{k^3}{k^2 + m_\pi^2} \sqrt{\frac{\pi\alpha^2}{kv}} I_{3/2}\left(\frac{kv}{2\alpha^2}\right), \quad (4.4)$$

where  $I_{3/2}$  is a modified Bessel function. The corresponding expression for the matrix element of the  $\pi a_0$  three-nucleon interaction (2.3) is obtained by the substitutions  $g_\sigma \rightarrow g_a$  and  $m_\sigma \rightarrow m_a$  and an overall change of sign in the expression (4.4), if the terms proportional to the total momentum of the intermediate nucleon in (2.3) are neglected. The sign change arises from different isospin dependence in (2.2) and (2.3).

The similarity in form between the  $\pi\sigma$  (2.2) and  $\pi\omega$  (2.5) three-nucleon interactions makes it obvious that the expression for the matrix element of  $V_{\pi\omega}$  is given by an expression of the form (4.4) with  $g_\sigma, m_\sigma$  replaced by  $g_\omega$  and  $m_\omega$ , and with an overall minus sign. The expression for the matrix element of the  $\pi\rho$  three-nucleon interaction  $V_{\pi\rho}^C$  (2.6) is obtained by the corresponding substitution in (4.4) of  $g_\sigma \rightarrow g_\omega, m_\sigma \rightarrow m_\rho$ , but without any change of the overall sign.

With the oscillator model density function (4.3) the matrix element of the  $N(1440)$  intermediate state  $\pi\sigma$  three-nucleon interaction (3.2a) takes the form

$$\begin{aligned} \langle V_{\pi\sigma}^* \rangle = & \frac{3}{\pi^4} \frac{g_\sigma g_\sigma^*}{m^* - m_N} \frac{f_{\pi NN} f_\pi^*}{m_\pi^2} \int_0^\infty dv e^{-v^2/2\alpha^2} \frac{v^2}{v^2 + m_\sigma^2} \\ & \int_0^\infty dk e^{-k^2/2\alpha^2} \frac{k^4}{k^2 + m_\pi^2} \sqrt{\frac{\pi\alpha}{kv}} I_{1/2}\left(\frac{kv}{2\alpha^2}\right). \end{aligned} \quad (4.5)$$

The corresponding expression for the matrix element of the  $\pi\omega$  exchange three-nucleon interaction  $V_{\pi\omega}^*$  (3.2b) can be obtained simply from (4.5) by means of the substitutions  $g_\sigma^* \rightarrow g_a^*$  and  $m_\sigma \rightarrow m_\omega$  and an accompanying overall sign change.

The harmonic oscillator model of a three-body bound state has special properties which allow a practical implementation of eq. (4.1) as shown above. Other techniques must be used for more realistic wave functions because the triton wavefunction depends on the initial and final Jacobi variables of the three nucleons. In general, the momentum transfer differences of eq. (4.2) must be supplemented by sums of Jacobi variables which, for

the harmonic oscillator, cancel the norm integral and therefore need not be considered in eq. (4.1). Further numerical calculations were made according to the methods described in [25] and [3b]. The  $S$ -state wavefunction that corresponds to the Malfliet-Tjon I-III interaction is given on a mesh over the coordinate space nucleon Jacobi variables described in [16]. After a Fourier transformation of the potentials of sections 2 and 3, the three dimensional integrals over two vector variables are straightforward to carry out numerically. On the other hand, in order to employ contemporary three-body wavefunctions from momentum space Faddeev calculations it is necessary to make a partial wave decomposition of  $V_3$  as was first done in [3b]. The final result of the partial wave decomposition of the potentials developed here is presented in Appendix A.

In Table I we give the numerical values for the matrix elements of the three-nucleon  $\pi$ -short range three-nucleon interactions that are associated with excitation of intermediate nucleon-antinucleon pairs. The numerical values have been given both for the case of the oscillator model wavefunction and the  $S$ -state wavefunction that corresponds to the Malfliet-Tjon I-III interaction [16]. The numbers in the table correspond to the case when the Bonn boson exchange potential model OBEPQ (Table V, [18]) parameterization has been used to construct the “effective” pion (pseudoscalar) and scalar and vector exchange potentials as described in section 2 above.

In Table II we give also for this parameterization of the TNI the numerical values for the Bonn OBEPQ and Paris wavefunction models — see Appendix for the technicalities involved — corresponding to the the  $\pi$ - $\omega$  and  $\pi$ - $\sigma$  exchange force only (according to Table I they dominate the overall  $\pi$ -scalar and  $\pi$ -vector exchange TNI). Table II confirms the trend of repulsion obtained with the more schematic wavefunction models used in the calculations of Table I. It also stresses once more the traditional extreme results of the OBEPQ potential for the triton binding energy.

All wavefunction models indicate that the  $S$ -state matrix elements of the  $\pi$ -short range three-nucleon interactions that are associated with intermediate  $N\bar{N}$  pair excitation are about 100 keV-200 keV (99 keV, 126 keV, 134 keV, 194 keV respectively for the H.O., MT, Paris and Bonn model wavefunctions), when these three nucleon interactions are constructed so as to

be consistent with the Bonn OBEPQ model for the  $NN$  interaction. This 100 keV has a substantial theoretical uncertainty margin that is due to the remaining uncertainty in the short range behaviour of the nucleon-nucleon interaction. To illustrate this we have also constructed these  $\pi$ -scalar and  $\pi$ -vector exchange three-nucleon interactions from the Paris model for the nucleon-nucleon interaction [20]. In that potential model the isospin independent scalar exchange component is much weaker than in other realistic phenomenological potential models, and is in fact repulsive at short range [12]. As a consequence that matrix element  $\langle V_{\pi\sigma} \rangle$  calculated with the oscillator wave function model is only 143 keV (or 113 keV for a even more consistent calculation with the Paris wavefunction) as compared to the corresponding value 392 keV obtained with the Bonn OBEPQ potential. This reduction of  $\langle V_{\pi\sigma} \rangle$  makes the net matrix element of the  $\pi$ -short range three nucleon attractive, when constructed from the parametrized Paris potential ( $\simeq -200$  keV for the oscillator wavefunction, or  $\simeq -101$  keV for the Paris wavefunction).

The difference of almost 300 keV (and a change of sign) between these two most consistent calculations presented here (a Bonn wavefunction and Bonn TNI compared to the Paris wavefunction and Paris TNI) may also in part be due to an inconsistency in the  $\mu$  parameter of the latter combination. The value of  $\mu$  labels an ambiguity in the relativistic corrections (ie from  $v/c$ -expansion methods) to operators involving pion exchange [26]. The same continuous parameter also acts as a chiral rotation to determine the pseudoscalar ( $\mu = 0$ ) or pseudovector ( $\mu = 1$ ) content of the pion-nucleon coupling in models of the nucleon-nucleon interaction or TNI's which obey approximate chiral symmetry [27]. Fixing arbitrarily this parameter (or the more general parameter  $\tilde{\mu}$  which includes a specification of energy transfer in the non-relativistic  $\pi NN$  vertex [28]) means choosing a special representation of a unitarily equivalent class of operators. Because of this unitary equivalence, the observables should not depend on the value of  $\tilde{\mu}$ , i.e. on the choice of representation, provided that all operators and wave functions are chosen consistently. The derivation of  $V_{\pi\sigma}$  of equation (2.2) and of  $V_{\pi\omega}$  of equation (2.5) corresponds to  $\tilde{\mu} = -1$ , as does the one-pion-exchange part of the Bonn OBEPQ. Thus it is completely consistent to determine the short range modifications of the simple meson exchange interactions of eqs. (2.2) and (2.5) by the corresponding components of the Bonn OBEPQ nucleon-

nucleon interaction model. The same replacement procedure with the Paris potential is, however, more problematic, as the parameter of that potential has been identified to be  $\tilde{\mu} = 0$  [28]. This inconsistency between the derivation of the TNI and the choice of short range modification is a possible cause of the discrepancy between our “consistent” calculations. Note that the harmonic oscillator NN model and the Malfliet-Tjon NN model do not have any pion exchange component so this particular consistency question does not arise. Also, the other parameter  $\nu$  of the unitarily linked operators of this paper is always  $\nu = 1/2$ , corresponding to no meson retardation, so no inconsistency can be attributed to  $\nu$ .

The TNI’s due to an  $N(1440)$  intermediate state have much smaller binding energy effects than those just discussed, a fortunate result for our goal of building TNI’s consistent with a given nucleon-nucleon interaction. The matrix element of the  $N(1440)$  intermediate state  $\pi\sigma$  three-nucleon interaction  $V_{\pi\sigma}^*$  (4.5) for the  $S$ -state oscillator model for the bound trinucleon states is 200 keV. Here we have used the parameters  $m_\sigma = 530$  MeV and  $g_\sigma^2/4\pi = 8.2797$  for the mass and (nucleon) coupling constants for the exchanged  $\sigma$  meson as suggested by the Bonn OBEPQ potential [18], and the short range form factors (3.7). The values of the meson- $NN^*$  coupling constants are those derived in section 3. The corresponding matrix element of the  $\pi\omega$  TNI  $V_{\pi\omega}^*$  (3.2b) is -150 keV so that the net contribution of the TNI associated with the  $N(1440)$  is repulsive and  $\simeq 50$  keV. This is about one half as large as the repulsive contribution that is due to the TNI associated with intermediate  $N\bar{N}$  pair excitation. When the same calculations are made with the substitution of the Malfliet-Tjon wave function for the oscillator wavefunction, the individual contributions are  $\simeq 65$  keV from  $V_{\pi\sigma}^*$  and  $\simeq -45$  keV from  $V_{\pi\omega}^*$ , for a total repulsion of  $\simeq 20$  keV. This total is again small compared to the repulsive contribution of  $\simeq 100$  keV that is due to the TNI associated with intermediate  $N\bar{N}$  pair excitation. Finally, in our most consistent calculation (OBEQ wavefunction and the Bonn boson exchange parametrization of the TNI), the total repulsion from the TNI associated with the  $N(1440)$  is about one third (+0.068:+0.194 keV) of that of the intermediate  $N\bar{N}$  pair excitation with the Bonn OBEPQ triton wavefunction.

In the  $S$ -state oscillator trinucleon model and the semirealistic, Malfliet-Tjon NN potential wave function and the more realistic Paris potential, the

combined contribution of all the  $\pi$ -short range TNI's considered here is thus about 150 keV, which corresponds to a significant fraction of the repulsive TNI contribution needed to explain the binding energies of the three-nucleon bound states. For the Bonn OBEPQ triton wavefunction this estimate is enhanced, as usual, to about 250 keV repulsion. To put these results in perspective we remind the reader that a similar evaluation of the  $\pi - \rho$  "Kroll-Ruderman interaction" of Ref. [4b] with the Malfliet-Tjon NN potential wave function yields about 340 keV repulsion when the formfactors are chosen similar to those of this paper.

This estimate of the repulsive contribution due to the  $\pi$ -short range three-nucleon interaction would a priori be expected to be altered by the  $D$ -state terms (or tensor correlations) in the trinucleon bound states. A suggestive estimate of these alterations is provided by the comparison of the "Kroll-Ruderman" interaction" used perturbatively with the simple Malfliet-Tjon NN potential wave function in Ref. [4b] and the expectation values of the same interaction with triton wave functions fully correlated by two- and three-nucleon interactions from Table IV of Ref. [8c]. One finds a reduction in the contribution of this nucleon-antinucleon pair term by 20% to 40% from the Malfliet-Tjon result when a careful calculation with the Nijmegen or Paris triton wavefunction is made. In the following section we attempt to understand these comparisons by showing (again with the aid of the harmonic oscillator trinucleon model) that the effect of the  $D$ -state admixture is a reduction of the pair term and enhancement of the isobar term so that effect on the net repulsive  $\pi$ -short range TNI is very small.



## 5. Estimate of the D-state contribution

The wavefunctions of the bound trinucleons that are obtained with realistic nucleon-nucleon interaction models contain  $D$ -state admixtures of the order 10%. As the  $\pi$ -short range three-nucleon interactions derived in sections 2 and 3 above have a rank-2 spatial tensor component they should be expected to have large  $SD$ -cross term matrix elements. In order to obtain a numerical estimate of the relative magnitude of this  $SD$ -state contribution we construct a model  $D$ -state wavefunction as [29]

$$\varphi_D = N_D \sum_{i < j} S_{ij} r_{ij}^2 \vec{\tau}_i \cdot \vec{\tau}_j \varphi_0 \phi_0, \quad (5.1)$$

where  $\varphi_0$  is the oscillator model trinucleon orbital wavefunction

$$\varphi_0 = \left( \frac{\alpha^2}{\pi\sqrt{3}} \right)^3 e^{-\alpha^2 r^2/4} e^{-\alpha^2 \rho^2/3}. \quad (5.2)$$

Here  $\vec{r}$  and  $\vec{\rho}$  are differences of the initial and final state nucleon Jacobi coordinates. The corresponding  $S$ -state wavefunction is simply  $\varphi_S = \sqrt{P_S} \varphi_0$ , where  $P_S$  is the  $S$ -state probability. If the three-body tensor terms as  $S_{12}S_{23}$  are dropped in the calculation of the  $D$ -state normalization factor  $N_D$ , it has the value

$$N_D = \frac{\alpha^2}{18\sqrt{5}} \sqrt{P_D}, \quad (5.3)$$

where  $P_D$  is the  $D$ -state probability.

As the main source of the  $D$ -state admixture is the pion exchange tensor interaction it is a natural approximation to evaluate the  $SD$  cross term matrix elements of the three-nucleon interactions with only the nucleon pair that involves a pion exchange having a  $D$ -state component. The relevant  $SD$ -cross term density function is then

$$g_D^{12}(\vec{\tau}, \vec{v}) = -\frac{\sqrt{P_D}}{18\sqrt{5}} S_{12}(\vec{\tau}) \vec{\tau}^1 \cdot \vec{\tau}^2 e^{-\tau^2/2\alpha^2} e^{-3v^2/8\alpha^2}, \quad (5.4)$$

where the tensor operator  $S_{12}(\vec{\tau})$  is defined as

$$S_{12}(\vec{\tau}) = 3(\vec{\sigma}^1 \cdot \hat{\tau})(\vec{\sigma}^2 \cdot \hat{\tau}) - \vec{\sigma}^1 \cdot \vec{\sigma}^2. \quad (5.5)$$

With this model density function the  $SD$ -state matrix element of the  $\pi\sigma$  TNI (2.2) can be expressed in the form

$$\begin{aligned} < V_{\pi\sigma} >_{SD+DS} = -\frac{\sqrt{P_D P_S}}{3\pi^4 \sqrt{5}} \frac{g_\sigma^2}{m_N} \left( \frac{f_{\pi NN}}{m_\pi} \right)^2 \\ & \int_0^\infty e^{-v^2/2\alpha^2} \frac{v^3}{v^2 + m_\sigma^2} \int_0^\infty dk e^{-k^2/2\alpha^2} \frac{k^3}{k^2 + m_\pi^2} \sqrt{\frac{\pi\alpha^2}{kv}} \\ & \left\{ 5vk I_{1/2}\left(\frac{kv}{2\alpha^2}\right) - 6\left[k^2 + \frac{v^2}{4}\right] I_{3/2}\left(\frac{kv}{2\alpha^2}\right) + vk I_{5/2}\left(\frac{kv}{2\alpha^2}\right) \right\}. \end{aligned} \quad (5.6)$$

The expressions for the  $SD$  matrix element of the  $\pi a_0$ ,  $\pi\omega$  and  $\pi\rho$  exchange three nucleon interactions that are associated with intermediate  $N\bar{N}$  pair excitation can be obtained by making the same substitutions in eq. (5.5) as described after eq. (4.5).

Assuming a 10%  $D$ -state admixture we find the numerical values for the  $SD$ -state matrix elements calculated using the expression (5.5) to be  $-80$  keV in the case of the  $\pi\sigma$  TNI and  $-20$  keV when the contributions of the  $\pi\sigma$ ,  $\pi a_0$ ,  $\pi\rho$  and  $\pi\omega$  three nucleon interactions are combined and constructed from the Bonn OBEPQ model for the nucleon-nucleon interaction [18]. The effect of the  $SD$ -state cross term matrix element is thus to reduce the repulsive  $\pi$ -short range contribution to the TNI that is associated with  $N\bar{N}$  pair excitation by about 20%.

The  $SD$  state matrix element of the  $\pi\sigma$  exchange interaction (3.2a) that is due to the excitation of intermediate  $N(1440)$  resonances can be derived by the same method using the  $SD$ -state density function (5.3). The resulting expression is

$$\begin{aligned} < V_{\pi\sigma}^* >_{SD+DS} = \frac{\sqrt{P_D P_S}}{6\pi^4 \sqrt{5}} \frac{g_\sigma g_\sigma^*}{m^* - m_N} \frac{f_{\pi NN} f_\pi^*}{m_\pi^2} \\ & \int dv e^{-v^2/2\alpha^2} \frac{v^2}{v^2 + m_\sigma^2} \int dk e^{-k^2/2\alpha^2} \frac{k^4}{k^2 + m_\pi^2} \end{aligned}$$

$$\sqrt{\frac{\pi\alpha^2}{kv}}\{[24k^2 - \frac{v^2}{2}]I_{1/2}(\frac{kv}{2\alpha^2}) - 24kvI_{3/2}(\frac{kv}{2\alpha^2}) + 6v^2I_{5/2}(\frac{kv}{2\alpha^2})\}. \quad (5.7)$$

The expression for the matrix element of the corresponding  $\pi\omega$  three nucleon interaction (3.2b) is obtained from (5.6) by the substitutions  $g_\sigma, g_\sigma^* \rightarrow g_\omega, g_\omega^*$  and  $m_\sigma \rightarrow m_\omega$  and a change of the overall sign. The numerical value for  $\langle V_{\pi\sigma}^* \rangle_{SD+DS}$  we find to be 94 keV, which is about one half of the corresponding  $SS$  matrix element. Addition of the matrix element of the  $\pi\omega$  exchange interaction  $\langle V_{\pi\omega}^* \rangle$  reduces this value to 21 keV. The net effect of the  $D$ -state is thus to increase the total matrix element of the  $\pi$ -short range TNI that arises from excitation of intermediate  $N(1440)$  resonances by about 40%.

As the numerical value for the  $SD$  state matrix elements of the  $\pi$ -short range three nucleon interactions that are due to excitation of intermediate nucleon-antinucleon pairs and  $N(1440)$  resonances are almost equal in magnitude and opposite in sign we conclude that the importance of the  $D$ -state admixture in the bound trinucleon states is very insignificant in this instance. Hence the estimates in section 4 that were obtained with pure  $S$ -state wavefunction models and which are very similar for the wavefunction models considered should be viewed as fairly robust.

## 6. Discussion

The present results demonstrate that the pion-scalar and pion-vector meson exchange three-nucleon interactions are important on the general scale of three-nucleon interactions. At the level of precision attained by calculations with the present realistic semiphenomenological nucleon-nucleon interactions, which also contain three-nucleon interactions, this TNI has to be included in the calculation of nuclear binding energies. The repulsive contribution of this  $\pi$ -short range exchange TNI appears able to explain most if not all of the repulsion hitherto ascribed to the purely phenomenological spin-independent TNI of short range, which was introduced to achieve agreement with the empirical binding energies of the few-nucleon systems [7].

The numerical values of the matrix elements of the  $\pi$ -short range three-nucleon interactions presented here should be realistic in spite of the employment of pure  $S$ -state models of the wavefunction of the bound three-nucleon system. By considering a schematic model for the  $D$ -state component it was shown that due to a number of cancellations the net effect of the  $D$ -state component for these TNI's is small.

The most uncertain in magnitude of the three-nucleon interactions considered here is that associated with the excitation of intermediate  $N(1440)$  resonances. The main uncertainty in this interaction is due to the unknown  $\omega NN^*(1440)$  coupling constant. Fortunately, this interaction has smaller effects on the triton binding energy than the interaction due to nucleon-antinucleon pair terms which can be directly related to realistic NN interactions in the manner shown here.

### Acknowledgements

The work of M.T.P. was supported in part by JNICT, under contract No. PBIC/C/CEN/1094/92 and "Contrato Plurianual", that of S.A.C. under NSF grant PHY-94081347 and that of D.O.R by Academy of Finland grant 7635. D.O.R thanks the Institute for Nuclear Theory, Seattle for hospitality at the time this work was completed. We thank Alfred Stadler for providing the Bonn OBEPQ and Paris three-body wavefunctions and the Iowa-Los Alamos three-body group for the Malfliet-Tjon wavefunctions.

## References

- [1] G. E. Brown and A. M. Green, Nucl. Phys. **A137** 1, (1969).
- [2] B. F. Gibson and B. H. J. McKellar, Few-Body Systems **3** (1988) 143; S. A. Coon and M. T. Peña, Few-Body Systems, **Suppl. 6**, 242 (1992).
- [3] S. A. Coon, M. D. Scadron, P. C. McNamee, B. R. Barrett, D. W. E. Blatt and B. H. J. McKellar, Nucl. Phys. **A317**, 242 (1979); S. A. Coon and W. Glöckle, Phys. Rev. **C23** 1790 (1981).
- [4] R. G. Ellis, S. A. Coon, and B. H. J. McKellar, Nucl. Phys. **A438**, 631 (1985); S. A. Coon and M. T. Peña, Phys. Rev. **C48** 2559 (1993).
- [5] C. R. Chen, G. L. Payne, J. L. Friar, and B. F. Gibson, Phys. Rev. **C33**, 1740 (1986); T. Sasakawa and S. Ishikawa, Few-Body Systems **1**, 3 (1986); J. Carlson, V. R. Pandharipande, and R. B. Wiringa Nucl. Phys. **A401**, 59 (1983).
- [6] S. A. Coon, J. Zabolitzky, and D. W. E. Blatt, Z. Physik A **281**, 137 (1977); R. B. Wiringa, Phys. Rev. **C43**, 1585 (1991); H. Kamada and W. Glöckle, Nucl. Phys. **A560**, 541 (1993).
- [7] R. Schiavilla, V.R. Pandharipande and R.B. Wiringa, Nucl. Phys. **A449**, 219 (1986).
- [8] A. Stadler, Bull. Am. Phys. Soc. **38**, 1013 (1993); A. Stadler, J. Adam Jr., J. Henning, and P. U. Sauer, “ $\pi$  and  $\rho$ -exchange Three-Nucleon Forces in the Three- Nucleon Bound State”, Proc. 14th European Conference on Few-Body Problems in Physics, Amsterdam, 23-27 August, 1993, Conference Handbook ISBN 90 5294 080 0, p. 192; A. Stadler, J. Adam Jr., J. Henning, and P. U. Sauer, “ $\pi$  and  $\rho$ -exchange Three-Nucleon Forces in the Three- Nucleon Bound State”, submitted to Phys. Rev C.
- [9] S. A. Coon and M. D. Scadron, Phys. Rev. **C23**, 1150 (1981); *ibid* **C42**, 2256 (1990);  $\pi$ -N Newsletter, **3**, 90 (1991).
- [10] J. Haidenbauer, K. Holinde, and A. W. Thomas, Phys. Rev. **C45**, 952 (1992); G. Janssen, K. Holinde, and J. Speth, Phys. Rev. Lett. **73**, 1332 (1994).

- [11] B.D. Keister and R.B. Wiringa, Phys. Lett. **173**, 5 (1986).
- [12] P.G. Blunden and D.O. Riska, Nucl. Phys. **A536**, 697 (1992).
- [13] M. R. Robilotta, Few-Body Systems, Suppl.**2** 2, 35 (1987).
- [14] J. Fujita and H. Miyazawa, Prog. Theor. Phys. **17**, 360 (1957).
- [15] O. Benhar, V. R. Pandharipande and S. Peiper, Rev. Mod. Phys. **65**, 817 (1993); R. B. Wiringa, ibid. 231. See also [5c] and [7].
- [16] J. L. Friar, B. F. Gibson, and G. L. Payne, Z. Phys. **A301**, 309 (1981); R. Malfliet and J. Tjon, Nucl. Phys. **A127**, 161 (1969).
- [17] A. Stadler, W. Glöckle and P. U. Sauer, Phys. Rev. C **44**, 2319 (1991).
- [18] R. Machleidt, K. Holinde and Ch. Elster, Phys. Rept. **149**, 1 (1987)
- [19] V. G. J. Stoks, R. A. M. Klomp, C. P. F. Terheggen, and J. J. de Swart, Phys. Rev. C **49**, 2950 (1994); M. M. Nagels, T. A. Rijken, and J. J. de Swart, Phys. Rev. D **17**, 768 (1978).
- [20] M. Lacombe et al., Phys. Rev. **C21**, 861 (1980).
- [21] J. M. Charap and M. J. Tausner, Nuovo Cim. **18**, 316 (1960).
- [22] M. Chemtob, J. W. Durso and D. O. Riska, Nucl. Phys. **B38**, 141 (1972).
- [23] T.-S. H. Lee and D. O. Riska, Phys. Rev. Lett. **70**, 2237 (1993); C. J. Horowitz, H. O. Meyer, and D. K. Griegel, Phys. Rev. C **49**, 1337 (1994).
- [24] T. Sasakawa, S. Ishikawa, Y. Wu, and T-Y. Saito, Phys. Rev. Lett. **68**, 3503 (1992).
- [25] S. A. Coon, M. T. Peña, R. G. Ellis, Phys. Rev. **C30**, 1366 (1984).
- [26] J. L. Friar, Phys. Rev. C **22**, 796 (1980).
- [27] S. A. Coon and J. L. Friar, Phys. Rev. C **34**, 1060 (1986); J. L. Friar and S. A. Coon, Phys. Rev. C **49**, 1272 (1994).

- [28] J. Adam, Jr., H. Göller, and A. Arenhövel, Phys. Rev. C **48**, 370 (1993);  
B. Desplanques and A. Amghar, Z. Phys. A **344**, 191, (1992).
- [29] E. Gerjuoy and J. Schwinger, Phys. Rev. **61**, 133 (1942).

## Appendix — Partial wave decomposition

While the calculations that employ the semi-realistic Gaussian and Malfliet-Tjon wave functions provide results that are qualitatively indicative, present state-of-the-art three-nucleon calculations have provided several examples of delicate cancellations that may be missed by schematic wave function models. This motivated a calculation based on more realistic wavefunctions, such as the ones obtained with the Paris and Bonn OBEPQ potentials (in references [8, 17]). These two potentials lead to quite different answers for the binding energy of the triton, and different probabilities for the S and D state components of the wavefunction.

The expectation value of the TNI can be calculated by means of equation (4.1). Although this equation is general, it requires the knowledge of  $g(\vec{\tau}, \vec{v})$ . Since this last function is not directly supplied by any standard Faddeev code, it demands extra computational effort. The algorithm that is usually employed with realistic interaction models is to evaluate the matrix element of the TNI in the partial wave decomposed basis used in the wave function calculation. The partial wave decomposition has been presented in a fairly general way in ref. [3b]. We here review its main steps, keeping the notation close to the one introduced in that work.

In the  $LS$  coupling scheme the notation for the partial wave decomposed wavefunction is

$$|pq\alpha\rangle_2 = |pq, [(l\lambda)L(s1/2)S]JJ_z, (t1/2)TT_z\rangle_2$$

where the index 2 specifies the particle that is taken to be the spectator in the definition of  $(\vec{p}, \vec{q})$ . To calculate the TNI matrix element in this basis, we start by separating explicitly the spin dependence of the TNI from the orbital one. For this purpose the TNI is decomposed in spherical components and consequently in spherical harmonics. Subsequently closed form quadrature over the angular arguments of these spherical harmonics is performed. This requires expansion in Legendre polynomials of the angle dependence of the form factors and propagators. These expansions (in three angles) are done numerically, through a Gaussian mesh of at least 20 points for each angle.



To illustrate the technique we consider the intermediate  $N\bar{N} \pi - \sigma$  TNI. For this particular case we have (leaving out the trivial isospin dependence),

$$(\vec{\sigma}^2 \cdot \vec{k}_\sigma)(\vec{\sigma}^1 \cdot \vec{k}_\pi) = \frac{4\pi}{3} k_\sigma k_\pi \sum_k \sqrt{2k+1} [\sigma(1) \times \sigma(2)]^k Y_{11}^k(\hat{k}_\pi, \hat{k}_\sigma). \quad (A.1)$$

Here the  $Y_{11}^k(\hat{k}_\pi, \hat{k}_\sigma)$  function stands for the two (coupled) spherical harmonics, which depend separately on the momenta of each meson, i.e.,

$$[Y^1 \times Y^1]^k = \sum_{m_1, m_2} C_{m_1 m_2 m}^{11k} Y_{m_1}^1(\hat{k}_\pi) Y_{m_2}^1(\hat{k}_\sigma). \quad (A.2)$$

After expressing the exchanged meson momenta in terms of the Jacobi coordinates  $\vec{p}, \vec{q}$  (for the initial three-nucleon state) and  $\vec{p}', \vec{q}'$  (for the final three-nucleon state),

$$\vec{k}_\pi = (\vec{p} - \vec{p}') - \frac{1}{2}(\vec{q} - \vec{q}'),$$

$$\vec{k}_\sigma = (\vec{p} - \vec{p}') + \frac{1}{2}(\vec{q} - \vec{q}').$$

the function  $Y_{11}^k(\hat{k}_\pi, \hat{k}_\sigma)$  can be decomposed in coupled spherical harmonics of simpler arguments:

$$\begin{aligned} Y_{11}^k(k_\pi, k_\sigma) &= 3 \sum_{r_1+r_2=1} \sum_{s_1+s_2=1} (-1)^{r_2} F(1, r_1, r_2) F(1, s_1, s_2) \left(\frac{1}{2}\right)^{r_2+s_2} \\ &\times \frac{|\vec{p} - \vec{p}'|^{r_1+s_1} |\vec{q} - \vec{q}'|^{r_2+s_2}}{k_\pi k_\sigma} \sum_{t_1 t_2} \left\{ \begin{matrix} r_1 & r_2 & 1 \\ s_1 & s_2 & 1 \\ t_1 & t_2 & k_1 \end{matrix} \right\} C_{000}^{r_1 s_1 t_1} C_{000}^{r_2 s_2 t_2} \\ &\times Y_{t_1 t_2}^k(\hat{\vec{p}} - \hat{\vec{p}}', \hat{\vec{q}} - \hat{\vec{q}}'), \end{aligned} \quad (A.3)$$

where

$$F(a, b, c) = \sqrt{\frac{(2a+1)!}{(2b)!(2c)!}}.$$

Denote by  $f_a(k_a)$  the product of the propagator of meson  $a$  by the two  $NNa$  couplings, including the form factor function that is introduced at the vertices. The momentum  $k_a$  depends on three angles: the angle between  $\vec{p} - \vec{p}'$

and  $\vec{q} - \vec{q}'$ , whose cosine is  $x_1$ , the angle between  $\vec{p}$  and  $\vec{p}'$ , the cosine of which is  $x_2$ , and the angle between  $\vec{q}$  and  $\vec{q}'$ , the cosine of which is  $x_3$ .

To prepare the angular integrations we can start by doing a decomposition in Legendre polynomials in the angular variable  $x_1$ :

$$f_\pi(k_\pi)f_\sigma(k_\sigma) = \sum_{l_1} \frac{2l_1 + 1}{2} g_{l_1}(|\vec{p} - \vec{p}'|, |\vec{q} - \vec{q}'|) P_{l_1}(x_1) \quad (A.4)$$

The two additional Legendre polynomial decompositions — in  $x_2$  and  $x_3$  — involve the function  $g_{l_1}(|\vec{p} - \vec{p}'|, |\vec{q} - \vec{q}'|)$  of Eq.(A.4) as well as powers of  $|\vec{p} - \vec{p}'|$  and  $|\vec{q} - \vec{q}'|$ , originated by contracting the function  $Y_{t_1 t_2}^k(\hat{\vec{p}} - \hat{\vec{p}}', \hat{\vec{q}} - \hat{\vec{q}}')$  from Eq. (A.3) with  $P_{l_1}$  from Eq. (A.4). The final result of the 3 decompositions is

$$\begin{aligned} H_{l_1 l_2 l_3 \alpha_1 t_3 \alpha_2 t_4}(p, p', q, q') &= \int_{-1}^1 dx_1 \int_{-1}^1 dx_2 \int_{-1}^1 dx_3 P_{l_1}(x_1) P_{l_2}(x_2) P_{l_3}(x_3) \\ &\quad \times |\vec{p} - \vec{p}'|^{\alpha_1 - t_3} |\vec{q} - \vec{q}'|^{\alpha_2 - t_4} f_\pi(k_\pi) f_\sigma(k_\sigma). \end{aligned} \quad (A.5)$$

Using Eq. (A.1), each term of the  $\sum_k$ ,  $V_{\pi\sigma}^k$ , gives for the orbital part of the matrix element,

$$\begin{aligned} \langle (l'\lambda') L' M' | V_{\pi\sigma}^k | (l\lambda) L M \rangle &= \frac{(4\pi)^2}{8} \hat{k}^2 \hat{L} \quad C_{\mu M M'}^{k L L'}(-)^{k+l+\lambda} \sum_{l_1 l_2 l_3} (-)^{l_1 + l_2 + l_3} \\ &\quad \times \hat{l}_1^2 \hat{l}_2^2 \hat{l}_3^2 \sum_{t_1 t_2 t_3 t_4} \hat{t}_1 \hat{t}_2 \hat{t}_3 \hat{t}_4 \quad \xi_{t_1 t_2 k}^{l_1 l_2 l_3 t_3 t_4} \left\{ \begin{matrix} t_2 & t_1 & k \\ t_3 & t_4 & l_1 \end{matrix} \right\} \left\{ \begin{matrix} l & l' & t_3 \\ \lambda & \lambda' & t_4 \\ L & L' & k \end{matrix} \right\} C_{000}^{t_1 l_1 t_3} C_{000}^{t_2 l_2 t_4} \\ &\quad \times \phi_{t_3 l_2 l'}(p, p') \phi_{t_4 l_3 \lambda \lambda'}(q, q'), \end{aligned} \quad (A.6)$$

with

$$\phi_{abcd}(k, k') = \sum_{f_1 + f_2 = a} F(a, f_1, f_2) k^{f_1} k'^{f_2} (-)^{f_2} C_{000}^{b f_1 c} C_{000}^{b f_2 d} \left\{ \begin{matrix} d & c & a \\ f_1 & f_2 & b \end{matrix} \right\} \quad (A.7)$$

and

$$\begin{aligned}
\xi_{t_1 t_2 k}^{l_1 l_2 l_3 t_3 t_4} &= \sum_{r_1+r_2=1} \sum_{s_1+s_2=1} C_{000}^{r_1 s_1 t_1} C_{000}^{r_2 s_2 t_2} F(1, r_1, r_2) F(1, s_1, s_2) (-)^{r_2} \\
&\times \left(\frac{1}{2}\right)^{r_2+s_2} \begin{Bmatrix} r_1 & r_2 & 1 \\ s_1 & s_2 & 1 \\ t_1 & t_2 & k \end{Bmatrix} H_{l_1 l_2 l_3 \alpha_1 t_3 \alpha_2 t_4}(p, p', q, q').
\end{aligned}
\tag{A.8}$$

( $\alpha_1 = r_1 + s_1$ ;  $\alpha_2 = r_2 + s_2$ ).

Applying the Wigner-Eckart theorem, we extract from Eq.(A.6) the orbital reduced matrix element, which ultimately we reconnect to the spin one, generating the desired final result.

**Table I**

	H.O.	MF I-III
$V_{\pi\sigma}$	0.392	0.457
$V_{\pi a}$	-0.012	-0.010
$V_{\pi\omega}$	-0.301	-0.337
$V_{\pi\rho}^C$	0.020	0.016
TOT	0.099	0.126

Matrix elements in MeV of the  $\pi$ -scalar and  $\pi$ -vector exchange interactions which involve an intermediate nucleon-antinucleon pair. The three-nucleon interactions are constructed from the Bonn boson exchange model OBEPQ for the  $NN$  interaction [18]. The matrix elements are obtained with  $S$ -state oscillator and Malfliet-Tjon I-III wavefunctions.

**Table II**

	PARIS	OBEPQ
$V_{\pi\sigma}$	0.551	0.836
$V_{\pi\omega}$	-0.417	-0.642
TOT	0.134	0.194

Matrix elements in MeV of the  $\pi$ - $\sigma$  and  $\pi$ - $\omega$  exchange interactions which involve an intermediate nucleon-antinucleon pair. The three-nucleon interactions are constructed from the Bonn boson exchange model OBEPQ for the  $NN$  interaction [18]. The matrix elements are obtained with the  $S$ -state components of the Paris and Bonn (OBEPQ) wavefunctions.

**Figure caption**

Fig. 1 (a)  $\pi$ -short range exchange three-nucleon interaction that involves an intermediate nucleon-antinucleon pair, (b)  $\pi$ -short range exchange

three-nucleon interaction that involves excitation of an intermediate  $N(1440)$  resonance. The wavy lines symbolize scalar and vector meson exchange.

This figure "fig1-1.png" is available in "png" format from:

<http://arXiv.org/ps/nucl-th/9503013v2>